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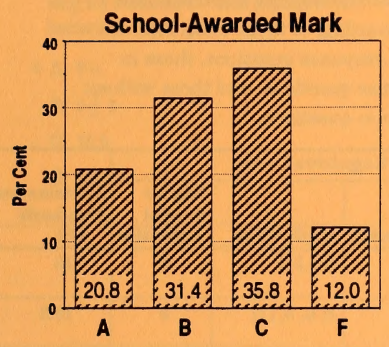
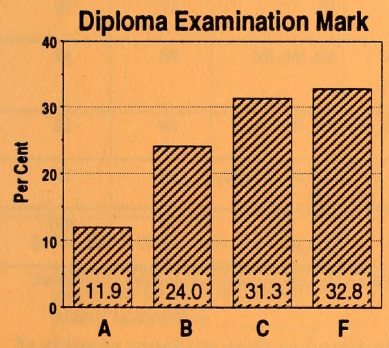
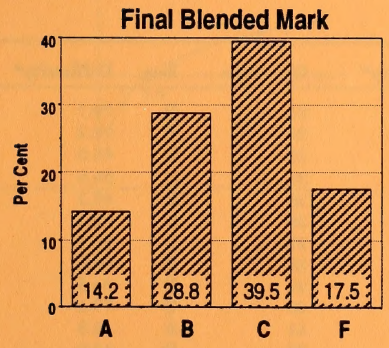
Mathematics 30

Diploma Examination Results
Examiners' Report for June 1992







The summary information in this report provides teachers, school administrators, students, and the general public with an overview of results from the June 1992 administration of the Mathematics 30 Diploma Examination. This information is most helpful when used in conjunction with the detailed school and jurisdiction reports that have been mailed to schools and school jurisdiction offices. An annual provincial report containing a detailed analysis of the combined January, June, and August results will be available this fall.

Description of the Examination

The Mathematics 30 Diploma Examination consists of three parts: a multiple-choice section of 40 questions worth 60%, a numerical-response section of 7 questions worth 10%, and a written-response section of 4 questions worth 30% of the total examination mark.

Achievement of Standards

The information reported is based on the final blended marks achieved by 10 749 students who wrote the June 1992 examination.

- 82.5% of these students achieved the acceptable standard (a final blended mark of 50% or higher).
- 14.2% of these students achieved the standard of excellence (a final blended mark of 80% or higher).

For the fourth year in a row, approximately 30% of the students who wrote the examination failed to achieve the acceptable standard. Students continue to have difficulty applying their knowledge of math in new contexts.

We continue to be concerned that a high number of students fail to meet the acceptable standard set by the examination whereas a comparatively low number of students fail to achieve 50% or higher on their school-awarded mark. This discrepancy suggests that provincial standards are not being consistently applied in classrooms throughout Alberta. Provincial standards for Mathematics 30 are explicitly set out in the *Mathematics 30 Diploma Examination Bulletin.*

Provincial Averages

- The average school-awarded mark was 65.7%.
- The average diploma examination mark was 57.8%.
- The average final blended mark, representing an equal weighting of the school-awarded mark and the diploma examination mark, was 62.1%.

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Subtest

When analyzing detailed examination results, please bear in mind that subtest results **cannot** be directly compared.

Results are in average raw scores.

Machine scored: 28.5 out of 46

Written response: 9.5 out of 20

Course Content

- Polynomial Functions: 4.3 out of 8
- Trigonometric and Circular Functions: 8.1 out of 13
- Statistics: 7.5 out of 13
- Quadratic Relations: 3.6 out of 8
- Exponential and Logarithmic Functions: 5.1 out of 8
- Permutations and Combinations: 4.1 out of 7
- Sequences and Series: 5.4 out of 9

Cognitive Levels

- Knowledge: 3.6 out of 5
- Comprehension: 9.3 out of 15
- Application: 21.6 out of 39
- Higher Mental Activities: 3.6 out of 7

Examination Blueprint

Each question on the examination is classified in two ways: according to the curricular content area being tested and according to the cognitive level demanded by the question. The examination blueprint illustrates the distribution of questions in June 1992 according to these classifications. Numbers in brackets [ ] indicate written-response questions, those in parentheses ( ) indicate numerical-response questions, and those without brackets or parentheses are multiple-choice questions.

	Questions by Cognitive Level				
Reporting Category	**Knowledge**	**Comprehension**	**Application**	**Higher Mental Activities**	**Examination Emphasis (%)**
Polynomial Functions		1	(1); [4]	2	12
Trigonometric and Circular Functions	3	4, 7; (2)	5, 6, 8; [1]	9	19.5
Statistics	15	16; (3)	10, 11, 12, 14; [3]	13	19.5
Quadratic Relations	18	(4)	[2]	17	12
Exponential and Logarithmic Functions	19	20, 21, 22	23, 24; (5)	25	12
Permutations and Combinations		26, 28, 32; (6)*	27, 30, 31	29	12
Sequences and Series	33	34, 38	35,36, 37, 39; (7)	40	13
Examination Emphasis (%)	7	22	60	11	100

*Numerical-response question 6 was deleted because ambiguous wording led students to incorrect strategies and hence incorrect answers.

This examination has a balance of question types and difficulties. It is designed so that students capable of achieving the acceptable standard will obtain a mark of 50% or higher and students capable of achieving the standard of excellence will obtain a mark of 80% or higher.

Multiple Choice

For a complete breakdown of student responses by alternative for the multiple-choice questions, please refer to the school and jurisdiction reports.

Students who enrol in Mathematics 30 are expected to be able to meet the acceptable standard. The standards set by the examination require students to successfully complete most of the knowledge, comprehension, and application questions. However, too many students are not successfully answering such questions. An illustrative question follows.

Question	Key	Difficulty*	Question	Key	Difficulty*
1	A	78.3	21	D	79.1
2	B	58.5	22	D	70.2
3	A	87.2	23	C	65.5
4	A	70.5	24	D	56.5
5	D	71.7	25	B	66.3
6	C	69.8	26	C	82.1
7	B	57.5	27	B	59.1
8	C	70.4	28	B	65.2
9	D	58.8	29	B	34.5
10	B	67.3	30	C	77.1
11	C	45.5	31	A	49.3
12	D	50.5	32	D	43.0
13	D	36.0	33	C	63.2
14	C	86.4	34	C	44.4
15	D	92.1	35	D	70.3
16	A	75.2	36	A	79.0
17	B	70.9	37	B	82.4
18	B	77.9	38	A	69.8
19	A	42.5	39	B	62.1
20	A	81.9	40	A	36.4

*Difficulty—percentage of students answering the question correctly

40. A culture of bacteria is being studied in a genetics experiment. The researcher observes that the bacteria double in number every 15 min. After 8 h, the number of bacteria in the culture is N. At this rate, how long will it take for the total population of bacteria to reach $16N$?

- **A.** 9 h
- **B.** $9\frac{1}{4}$ h
- **C.** 16 h
- **D.** 128 h

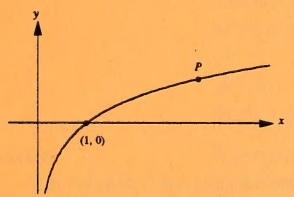
Question 40 reinforces the notion that students should check an answer to ensure that it is reasonable. This question required students to recognize that bacteria doubling is an example of exponential growth or a geometric sequence with a common ratio of 2. Approximately 31% of the students who wrote the examination interpreted this doubling as a linear function and chose alternative D. If students had examined the reasonableness of this solution, they would have discovered that after 128 h there would be an incredibly large number of bacteria. An almost equal number of students, 36.4%, recognized the doubling process as a geometric sequence and therefore chose the correct alternative.

Numerical Response

Question	Key	Difficulty*
1	11.0	58.5
2	3.8	49.2
3	5.3	39.0
4	5.0	21.9
5	2.91	47.2
6	—	—
7	8.4	33.0

*Difficulty—percentage of students answering the question correctly

5. The sketch of the graph of $y = \log_2(x)$ is shown below. If $P(x, 1.54)$ is a point on this graph, then the value of x correct to the nearest hundredth is _____.

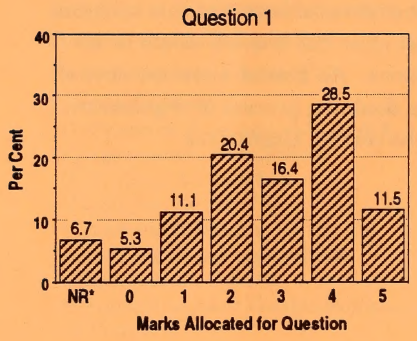


Question 5 required students to use their knowledge of logarithmic equations to determine the value of x in the equation $y = \log_2(x)$, given the value of y. Students were provided with the graph of $y = \log_2(x)$ to illustrate the relationship between solving a logarithmic equation and a point on the graph. We expect that students who achieve the acceptable standard can solve simple exponential and logarithmic equations and can use the graphs of logarithmic functions to estimate the value of one of the variables, given the other variable. Only 42.8% of all students who met the acceptable standard were able to solve this logarithmic equation. Of those students who met the standard of excellence, 82.7% were able to complete this question correctly.

Written Response

Questions in the written-response section dealt with four of the seven content strands for Mathematics 30. Students performing at the acceptable standard were expected to obtain at least half marks on all questions. Students performing at the standard of excellence were expected to answer all questions almost perfectly. We were very pleased that 83.8% of students attempted all the

written-response questions. Having attempted the questions, most students were able to receive some marks. Only 12.0% of all students received no marks for question 1, 22.6% received no marks for question 2, only 10.5% received no marks for question 3, and only 13.1% received no marks for question 4.



*NR—No Response

Question 1 required students to describe the effects of the parameters a, c, and d on the graph of $y = \sin \theta$. To describe these effects, students used a variety of methods: paragraph form, point form, and diagrams. All these methods were acceptable. We expected that students who met the acceptable standard would be able to describe the effects of each parameter on the graph. Students who were able to do this received 4 out of the 5 marks. Of the students who achieved the acceptable standard but not the standard of excellence, 33.1% met the expectation for this question. Those students who achieved the standard of excellence were expected to discuss not only the effects of each parameter but also the effects of these parameters on the domain or range of the function. Of the students who achieved the standard of excellence, only 34.1% met the expectation for this question.

On this 5-mark question, the average mark was 2.73 or 54.6%.

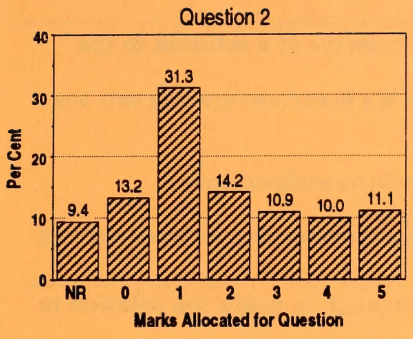
Scoring Question 1: To receive 6 check marks, students needed to describe the parameter, either explicitly or implicitly, and then its effect on the graph of $y = \sin \theta$. To receive 7 or 8 check marks, students needed to discuss the overall effect of all three parameters.

Checks	Marks
7 or 8	5
6	4
5	3
3 or 4	2
1 or 2	1

One example of a response that would receive 5 marks is:

> The parameter $\frac{1}{2}$ is the amplitude for the graph. The amplitude changes from 1 to $\frac{1}{2}$. The parameter $+\frac{\pi}{4}$ moves the graph of $y = \sin \theta$ left $\frac{\pi}{4}$ units. The parameter -2 moves the graph down 2 units. These three parameters change the range of $y = \sin \theta$ from $-1 \le y \le 1$ to $-\frac{5}{2} \le y \le -\frac{3}{2}$.

Note: In scoring this question, markers found that students had a difficult time expressing their ideas. They misused phrases and terms such as "altitude" for "amplitude;" they said that the curve became "flatter," that the graph moved "left $-\frac{\pi}{4}$ units" or "2 translations" down, and that "-2 changes the x-axis. The x-axis is now at -2." Students need to continue to practise writing *about* mathematics.



Question 2 required students to solve a problem using their knowledge of quadratic relations. The question postulated that two people had started to graph a conic. Students needed to analyze the information provided, identify unknown information, decide on a strategy to solve the problem, and carry through to complete the graph of the conic. One way of solving this problem was to identify the eccentricity of the conic, determine that the conic was a hyperbola, and then draw a hyperbola. Students who achieved the acceptable standard were expected to identify the eccentricity, determine the conic, and then draw at least one branch of the hyperbola. Only 35.3% of these students met or exceeded this expectation. Students who achieved the standard of excellence were expected to realize that a hyperbola has two branches and to draw the two branches. In fact, 73.1% of these students did recognize that a hyperbola has two branches. However, 25.9% of the students who met the standard of excellence misinterpreted the directrix to be the axis of symmetry.

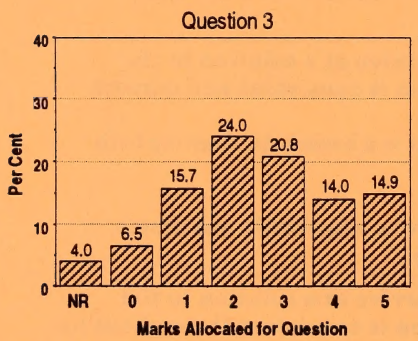
On this 5-mark question, the average mark was 1.88 or 37.6%.

Scoring Question 2: A scoring guide was used to score question 2. Markers reviewed each student's response and matched the response to the criteria.

Scoring Guide for Written-Response Question 2

Score	Criteria
5 marks	**Identifies a strategy, carries it through to arrive at a solution to the problem, and then confirms that the solution is consistent and correct throughout by:** calculating eccentricity = 2; stating that the conic is a hyperbola; drawing both branches of the hyperbola correctly ***or*** stating that the conic is a hyperbola; explaining why a hyperbola was chosen; drawing both branches of the hyperbola correctly
4 marks	**Identifies a strategy, carries it through to arrive at a solution to the problem, and then confirms that the solution is consistent throughout by:** calculating eccentricity = 2; stating that the conic is a hyperbola; attempting to draw two branches of the hyperbola and drawing only one branch correctly ***or*** calculating eccentricity = 2; stating that the conic is half a hyperbola; drawing one branch correctly ***or*** stating that the conic is a hyperbola; explaining why a hyperbola was chosen; drawing two branches but only one branch correctly

3 marks	**Identifies a strategy and carries it through to arrive at a solution to the problem by:** calculating eccentricity = 2; stating that the conic is a hyperbola; drawing only one branch of the hyperbola correctly ***or*** completing the graph of the hyperbola correctly with no explanations ***or*** attempting to calculate eccentricity = 2; stating the corresponding conic; attempting to draw the corresponding conic
2 marks	**Identifies a strategy and begins to carry it through to begin the solution to the problem by:** calculating eccentricity = 2; stating an incorrect conic (for example, parabola); drawing one branch of the hyperbola correctly ***or*** stating some value for the eccentricity; stating the corresponding conic ***or*** calculating eccentricity = 2; drawing one branch of the hyperbola correctly ***or*** drawing two branches of the hyperbola, only one correctly, with no explanation
1 mark	**Identifies a strategy to solving the problem by:** stating that a hyperbola is being drawn ***or*** drawing a reasonable representation of one branch of a hyperbola ***or*** calculating eccentricity



Question 3, part a, required students to determine a 90% confidence interval for the percentage of yeses in a population, given the number of yeses in a sample. Part b required students to describe the significance of the 90% confidence interval from part a. Part c asked how the confidence interval would change if the sample size increased. Students who achieved the acceptable standard were expected to successfully complete parts a and b and to begin a description in part c. In fact, 56.7% of the students who achieved the acceptable standard but not the standard of excellence met or exceeded this expectation. Students who achieved the standard of excellence were expected to score at least 4 out of 5 marks on this question, that is, to successfully complete parts a, b, and c. Of the students who achieved the standard of excellence, 69% met this expectation. Markers felt that students need to be given more opportunity to discuss the meaning of confidence intervals. Students need to realize that a confidence interval is obtained after several surveys are conducted: if a survey is replicated 100 times, you can expect similar results 90 times; however, results will be different 10 times. Many students interpreted the confidence interval to mean that 30 to 50 people are expected to respond "yes." (For additional information about how students misunderstood confidence interval, refer to

"Directions for the 1993 Examinations" in the *Mathematics 30 Diploma Examination Bulletin, 1992-93 School Year.*)

On this 5-mark question, the average mark was 2.57 or 51.4%.

Scoring Question 3: Part a was worth 1 mark. Students were awarded the mark for part a if they indicated that the 90% confidence interval was 30–50% ***or*** 40% ± 10%. Students were also awarded the mark if they extrapolated the interval from the 90% Box Plots to arrive at a symmetric interval about 40% where the lower limit was greater than 25% and the upper limit was less than 55%.

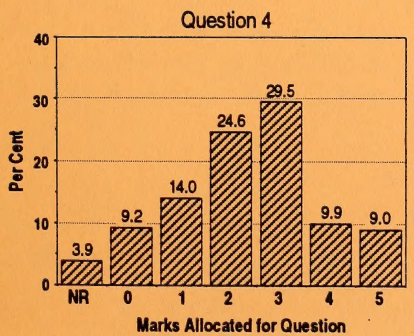
Part b was worth 3 marks, which were determined as follows:

Score	Criteria
3 marks	Student's response indicates that a level of certainty or uncertainty exists and that the proportion of yeses in the population will be within the confidence interval identified in part a; the response clearly communicates these two criteria.
2 marks	Student's response indicates that a level of certainty or uncertainty exists and that the proportion of yeses in the population will be within the confidence interval identified in part a; the response does not clearly communicate these two criteria.
1 mark	Student's response indicates a level of certainty or uncertainty ***or*** that the proportion of yeses in the population will be within the confidence interval identified in part a.

One example of a response that would receive 3 marks is:

> We are 90% confident that between 30–50% of the population of bus commuters would respond "yes" if they were asked whether or not their bus service was adequate.

Part c was worth 1 mark. Students were awarded the mark if their response indicated that the confidence interval would become shorter or that the margin of error would decrease if the sample size increased.



Question 4 required students to use their knowledge and understanding of polynomial functions to analyze the graph provided. Students who met the acceptable standard were expected to correctly identify the zeros of the polynomial function and the lowest possible degree of this polynomial. Of the students who achieved the acceptable standard but not the standard of excellence, 57% met or surpassed this expectation. Students who achieved the standard of excellence were expected to complete part c of the question by identifying other possible degrees for the

polynomial and explaining their answer. Only 59.5% of the students who achieved the standard of excellence scored at least 4 out of 5 marks on this question.

On this 5-mark question, the average mark was 2.36 or 47.2%.

Scoring Question 4

Part a was worth 2 marks, which were determined as follows:

Score	Criteria
2 marks	Student's response indicates that the zeros of the polynomial are –5, 0, 4 ***or*** –5, –5, 0, 4.
1 mark	Student attempts a solution to the problem by writing the solution as factors $x(x + 5)^2(x - 4)$ ***or*** by listing 3 distinct factors: x, $(x + 5)$, and $(x - 4)$ ***or*** by identifying 2 distinct zeros such as –5 and 4.

Part b was worth 1 mark. Students were awarded the mark if they identified the lowest possible degree of the polynomial as being 4.

Part c was worth 2 marks, which were determined as follows:

Score	Criteria
2 marks	Student's response clearly indicates other possible degrees (6, 8, . . . or any even degree larger than 4) for the polynomial and explains the reasoning for selecting that number of degrees.
1 mark	Student's response indicates other possible degrees for the polynomial or explains why there are other possible degrees for this polynomial.

One example of a response that would receive 2 marks is:

> This polynomial could be degree 6, 8, 10, 12 . . . . A polynomial function that could describe this graph is $P(x) = ax(x - 4)(x + 5)^2$. The factor that can have a higher degree without changing the shape of the graph is $(x + 5)^2$. This factor could have degree 4, 6, 8 . . . . As long as this factor has an even degree, the graph will continue to "touch" the x-axis at –5. If this factor has a higher even degree, the polynomial $P(x)$ will also have a higher even degree.

For further information, contact Florence Glanfield, Lowell Hackman, or Phill Campbell at the Student Evaluation Branch, 427-2948.